Activity: Caged Mice Problem

Name of Lesson: Caged Mice Problem

Mathematics: Part-Whole relationships, properties of operations (addition), partitioning, composing and decomposing

Grades: K-2

Standards for Mathematical Practices

- Making sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

Time Needed Several class periods

Materials Needed
Two color counters, unifix cubes, tiles, or other manipulative for counting, double number lines, dominoes, math journals, MVP cube #3, #7, and #8

Vocabulary / Visual Models
Unknown, word problem, equation, equal, addition, large cage, small cage, connected, turn-around facts, conjecture, odd, even
Visual Models: two color counters, cubes, tiles or some kind of counter, double number lines, dominoes

Procedures:
Launching The Lesson

1. Write the following statement on the board:

Mathematicians talk to and question other mathematicians in order to help them understand.

Read the statement about what mathematicians do. In today's lesson, you will be working with another friend who you can talk to and ask questions about ideas you are confused about. You will be helping each other make sense of the math that you do.

2. Present the following problem to the students. Ask the students to do a "cold read" of the problem. (During a "cold read" the student reads the passage with no help, circling any words that are troublesome or confusing. It is okay if the student struggles; as these errors will become a part of the following discussion.)

The problem can be written on a class chart for reference during the time students are exploring the problem for solutions.

> ***Ricardo has 7 mice. He keeps them in two cages that are connected so that the mice can go back and forth between cages. One of the cages is big and one is small. Show all the ways that 7 mice can be in two cages.***

3. Create the four-box graphic organizer attached to this lesson on large chart paper. Asks students the four guiding questions below and record their ideas in the four boxes on your chart.

Guiding Questions:

- What is this problem mostly about? (Mice that live in cages)
- Are there any words in the problem that are confusing to you? (List any words the students bring up and briefly discuss and clarify the meaning by asking the students to help you.) What facts do we know from the problem? (7 mice, 2 cages that are connected - one big and one small)
- What are you being asked to do? (Show all the ways 7 mice can be in two cages.)

4. If the students need any clarification before they begin to work on the problem ask, "Who can tell me one way that the mice might be in the two cages?" Write down the student solution, asking the class if they agree or disagree with the example being a possibility. If needed, ask, can anyone else think of another possibility? Record the solution and ask if this is another possibility?

Often this problem needs some clarification. The easiest way to clarify the problem is to do one or two solutions only as a group so students understand what is expected.

Remember, if someone asks if there can be 0 mice in one cage, refrain from answering this question. As the teacher/facilitator, you do not want to give answers that children can investigate and discover those ideas.

Exploring the Problem
1. Working with a partner, have students work to find and record all the ways the 7 mice can be partitioned into the 2 cages.

2. Allow students to use whatever manipulatives they need to solve the problem, using their math journals to show their findings.

3. Probe student thinking using the questions below if a student gets stuck or needs to go deeper.

4. As you move around the classroom, look for different strategies that students are using as well as different ways they are organizing their work. Select 3-4 student work samples that can be used to share <u>solutions</u> and <u>organizational strategies</u>.

Guiding Questions:
If students get stuck, ask:

- What do you need to find out?
- Would it help to draw a picture?
- How would you describe the problem in your own words?
- What tools would help you?
- How have you tackled similar problems?
- What about putting your ideas in order?
- Explain what you've done so far. What else is there to do?
- Why did you decide to use this method over another way?

Summarizing the Learning
1. Gather the students at your meeting area to present and discuss solutions to the problem. Use the MVP Cube #3 questions to start the conversation.

Adapted from Carpenter, Thomas, et. al.; Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School; Heinemann; 2003; pgs. 65-70. and guiding questions from PBS Mathline bookmark.

2. During the summarizing stage, it is appropriate for the teacher to model how to use the t-chart as an organizational tool to record the combinations shared by the students.

3. Move toward discussing the idea that there are "pairs of numbers" and it IS different when there are 3 mice in the large cage and 4 mice in the small one, than 4 mice in the large cage and 3 mice in the small one.

Figure 1

Big Cage	Little Cage
0	7
1	6
2	5
3	4
4	3
5	2
6	1
7	0

4. Review again to see if the group discovered all the possibilities. Ask, How do you know you found all the possible ways (combinations)? (There should be 8 arrangements. All the one-digit numbers from 0 – 7 are represented in the listed combinations. See Figure 1.)

5. Use some of the guiding questions to target student thinking at a more focused and rigorous level.

Guiding Questions:
To encourage reflection, ask:

- How did you get your answer?
- What strategy/ies did you use?
- What ideas have you used or learned before that were useful in solving this problem?
- Will your strategy work for similar problems? Explain your thinking?
- Is there a more efficient strategy?
- Why did you decide to organize your results like that? Would this strategy work with other numbers?

6. This is a good stopping place for this lesson. When you return to the chart at your next math meeting, ask the students to review what the problem was about and what the data shows. Tell the students that you want them to study the class chart carefully. Ask, **What patterns do you notice?**

The discussion that follows can lead to finding patterns in the chart, making conjectures to test out and prove or disprove, or the discussion might lead to related investigations for other numbers.

As the students share what they notice, record their words on the chart paper. When you model like this, you are helping students see how their words "land on the paper." These words have to make sense and give evidence of what they notice and think.

Figure 1	
Big Cage	Little Cage
0	7
1	6
2	5
3	4
4	3
5	2
6	1
7	0

- What patterns do you notice? (The numbers start at 0 and go up by one for the big cage and they go down by one for the small cage. Some of the combinations are the same numbers. The two numbers are just turned around.)

- **Ask: What do you notice about these turn-around number pairs? Do they represent the same quantity?** Model the student's ideas on a double number line. For example 2 + 5 and 5 + 2. Starting at 0, use the top of the line to jump 5 and then jump 2. Then use the bottom of the number line starting at 0 to jump 2 and then 5. This visual model is a great tool for how students can use a model to think about and prove their ideas.

- **Ask: Do you think this idea will work with other numbers as well? If so, how can we test our ideas and what rule could we make about these pairs of numbers?**

- Have the students dictate a sentence about what they believe to be true about turn-around number pairs when added together. Call this statement a conjecture. *Note: Students do not have to connect the math term.*

    Ex. When you add two numbers, you can change the order of the numbers you add and you will still get the same number.

- **What visual models could you use to prove your conjecture? How many ways can you show it?** (Two color counters, dominoes, double number lines)

  Student learn through multiple opportunities to use multiple representations before the conjecture becomes a generalization. See "Extending the Lesson" to investigate these visual models as a way to justify the conjecture stated above. This investigation can carry over to another day or days. This is not a bad thing. The students are engaged in meaningful math investigations, collecting more data, and learning how to think like mathematicians.

Adapted from Carpenter, Thomas, et. al.; Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School; Heinemann; 2003; pgs. 65-70. and guiding questions from PBS Mathline bookmark.

Extending the Learning

1. Investigate ways to justify the conjecture the class wrote in the lesson, (when you add two numbers, you can change the order of the numbers you add and you will still get the same number ($a + b = b + a$). By using a variety of visual models, such as dominoes, double number lines, and two color counters students justify their conclusions.

2. Students then come together to share and justify their conclusions. It is productive to ask children whether their conjectures are always true for all numbers and how they know. When summarizing this investigation, use the MVP Cubes #7 and #8 to start the conversation. This lays the groundwork for developing these ideas in more depth in later grades.

3. Another investigation students could explore: What happens if I change the 7 to 9 mice? To 10? 15? What statement can we make? Make a conjecture for any number of mice. Is this always true for all numbers and how do you know? What visual models can you use to justify your thinking?

Scaffolding Suggestions for Differentiation of Content, Product, and/or Process

Content:

To build awareness of more than and less than, pose another problem.

If cage A always had more mice than cage B, what combination of mice could cage A and Cage B have?

Choose a number of mice appropriate for your level.

Product:

To prove a conjecture and to make a "rule", students must experience multiple opportunities to collect data to determine if what they notice always works.

Process:

Using 2 hula-hoops labeled large and small, have students act out the different representations to fill the cages with 7 mice.

Adapted from Carpenter, Thomas, et. al.; Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School; Heinemann; 2003; pgs. 65-70. and guiding questions from PBS Mathline bookmark.

Assessment of the Focused Standards

Caged Mice Performance Task (see attachment)

Sara and Bob are trying to determine different ways they can make two pizzas. They have 9 pieces of pepperoni and two pizza crusts. Sara explains that her pizza can have 5 pieces of pepperoni and that Bob's can have 4 pieces of pepperoni. Bob explains another way they can use the pepperoni is to put 5 pieces on his pizza and 4 pieces on Sara's pizza. Sara disagrees, with Bob and says that would not work. Who do you agree with and why?

Represent your thinking with a visual models, numbers, and words.

What to look for:

- Are the students able to identify 5 + 4 as the same as 4 + 5?
- Are the students able to explain that Bob's idea is like Sara's because they are the same numbers that equal the same amount?
- If students are able to recall facts on the second question, are they able to explain their thinking and understanding of the community property in question 1? If not, then are these facts "memorized" with little understanding of relationships between numbers?

<u>Teacher Reflection:</u>

- What went well? What didn't go well?
- What changes or adaptations would you make to this lesson?
- What surprises or "ahas" did you have? Explain.
- What misconceptions did your students have?
- What connections did you and your students make?
- What Mathematical Practices were best represented?
- How will you assess understanding of the concepts in this lesson?

Name: ____ Caged Mice Performance Task

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Represent your thinking with a visual models, numbers, and words.

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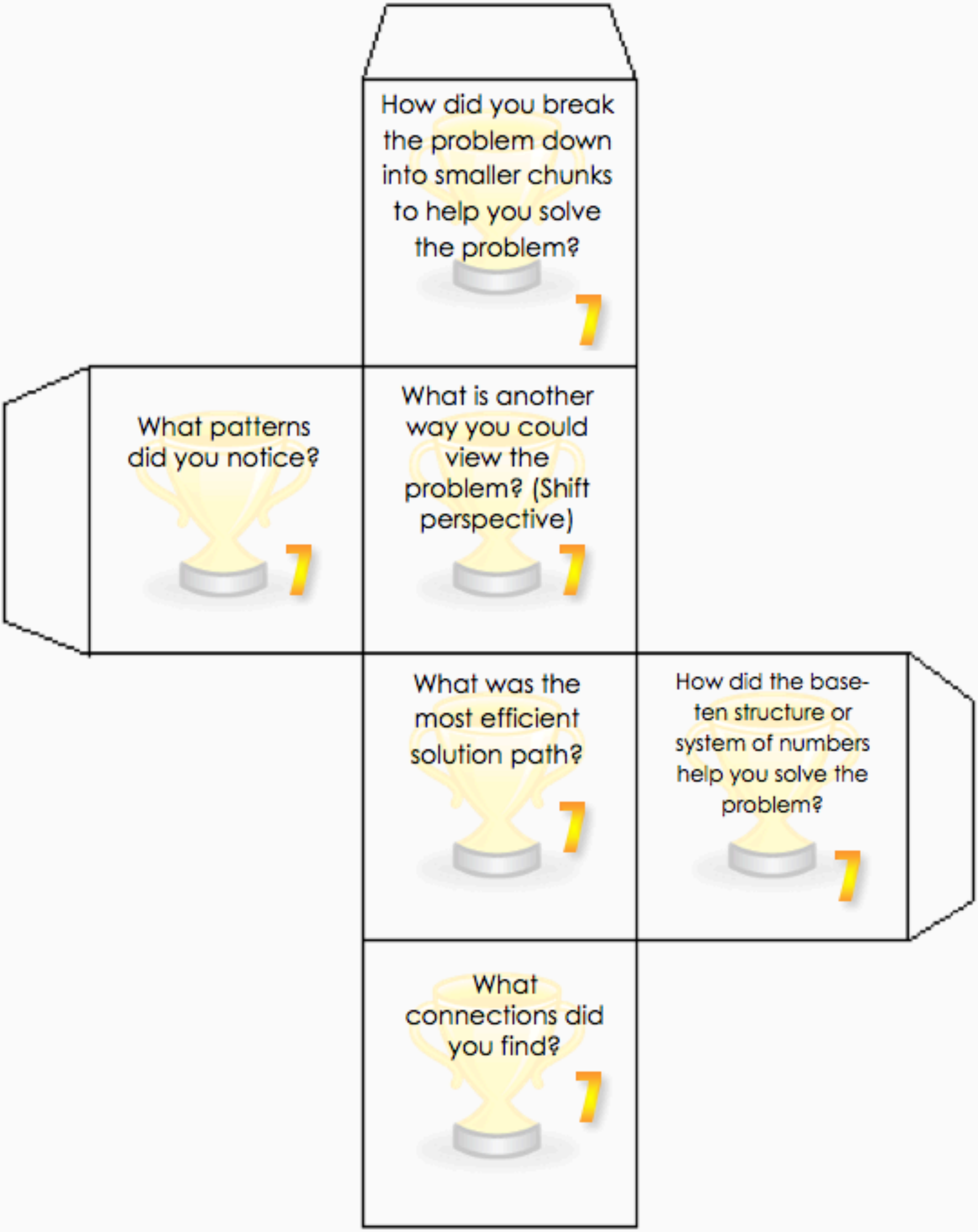
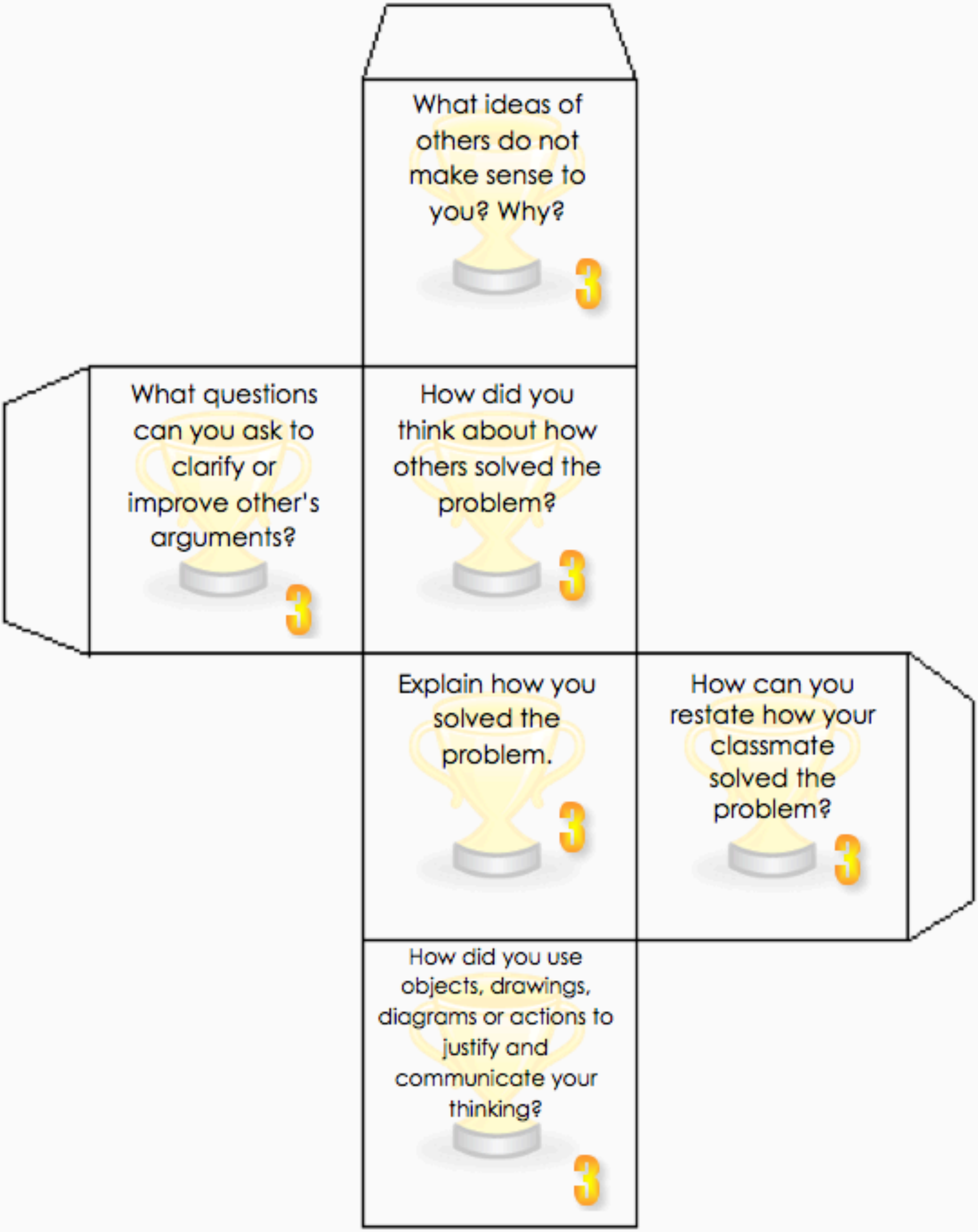
What is another solution that Sara and Bob could do with their nine pieces of pepperoni? Justify your thinking with numbers, visual models, and words.

K-2 IMPACT
Investigating Mathematical Practices and Classroom Teaching

What is the problem mostly about?	Vocabulary
What facts can you list from the problem?	**What is the problem solving?**

Teaching & Learning Collaborative


How did you break the problem down into smaller chunks to help you solve the problem?
What patterns did you notice?
What is another way you could view the problem? (Shift perspective)
What was the most efficient solution path?
How did the base-ten structure or system of numbers help you solve the problem?
What connections did you find?


What ideas of others do not make sense to you? Why?
What questions can you ask to clarify or improve other's arguments?
How did you think about how others solved the problem?
Explain how you solved the problem.
How can you restate how your classmate solved the problem?
How did you use objects, drawings, diagrams or actions to justify and communicate your thinking?
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